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Comparison of Analytical and Computational Vibration Analysis of Elliptical Plates with Fully Clamped Boundary Condition

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Abstract— Determination of mode shapes and natural frequencies is important to design the structural components like plates, shells and beams which are generally subjected to vibration. Vibration of plates has been the study of importance due to its wide engineering applications such as aerospace, marine, and ship industries. Especially in aerospace and ship industries, it is observed that various shapes of plates do occur. Elliptical plates are used in order to avoid the high stress concentration. In recent years, the vibration, buckling and bending problems of elliptical plates subjected to different types of loads such as, the combined action of lateral load and in-plane force are being studied. In this project, vibration analysis is done on elliptical plates for different material properties using both analytical and computational analysis. The comparison of the modal behavior by using these two analyses is done.

Index Terms— Clamped Condition, Hyper Mesh, Modal Analysis, Mode Shapes, 3D Meshing, Orthogonal Polynomials, Rayleigh Ritz Method.

1 Introduction

Orthogonal Polynomials have been emerging as powerful polynomials in recent years for the study of Vibration of Plates. In recent times, lot of research has been carried out on vibration analysis of plates using 2 Dimensional orthogonal polynomials. By using orthogonal polynomials in Rayleigh Ritz method, Singh B and Chakraverty S [1] have found the frequencies of the first few modes of elliptical plates subjected to transverse vibrations. By varying thickness linearly with different aspect ratios, Saleh M. Hassan [2] has obtained the first four frequency parameters of elliptical plates by considering different boundary conditions on either side of the plates. K.M. Liew and K.Y. Lam [3] have presented a paper on flexural vibration of skew plates. T P Chang and M H Wu [4] have published a paper on anisotropic plates with mixed boundary conditions and concentrated masses. Rajalingham C, Bhat R.B have published a paper on Vibration of Clamped Plate using exact modes of circular plates using as shape functions.T Lakshmi Reddy, P V Pavan Kumar, Akshay [7] have proposed a paper on modal analysis on plates using Ansys.

In this paper, we are doing Vibration analysis on elliptical plates for fully clamped boundary condition. For Computational analysis we are using Ansys as a tool for the modal analysis. Analysis is done on two different materials i.e., Steel 304L and Aluminium 6063L. The results have been compared for both types of analysis for two materials.

Analytical Method: This method consists of three steps; Generation of Orthogonal Polynomials over the ellipse domain in x-y plane and satisfying boundary conditions by using Gram Schmidt method. The second step is to convert the problem into the standard Eigen value problem by minimizing the orthogonal polynomials in the Rayleigh's Ritz method. The third and final step involves finding the solution of this Eigen value problem, thus obtaining the frequencies for fully clamped boundary condition.

Computational Method: This method consists of three steps; Creating Solid model of ellipse shape with aspect ratio of 0.5.

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Meshing and applying boundary conditions using Hypermesh and finally carried out modal analysis for natural frequencies using Ansys.

2 Analytical Method

2.1 Generation of Orthogonal Polynomials

We follow the same approach as used by Bhat [1] and Liew et al. [3] for a rectangular plate. For the elliptic plate we start with the linearly independent set

$$\{F_i(x,y)=u^2 f_i(x,y)\}_{i=1}^{n} \quad (1)$$

where x, y is a point in the domain R occupied by the plate with boundary ∂ R defined by

$$x^2/a^2+y^2/b^2=1. \quad (2)$$

The variable u which varies from 0 at the boundary to 1 at the centre of the plate is defined by

$$u=1-x^2/a^2-y^2/b^2 \quad (3)$$

The functions f;(x, y) are suitably chosen linearly independent functions. Note that each F_i should satisfy the clamped boundary condition. The inner product of two functions f and g is defined by

$$\langle f,g\rangle=\int_R\int f(x,y)g(x,y)dxdy\ , \quad (4)$$

and the norm of a function f by

$$||f||=\sqrt{\langle f,f\rangle}=\left[\int_R\int f^2(x,y)\,dxdy\right]^{0.5} \quad (5)$$

The orthogonal functions $\Phi_i(x, y)$ are generated by the Gram Schmidt process, the algorithm for which may be summarized as follows:

$$\Phi_i=F_i$$

$$\Phi_i=F_i-\sum_{j=1}^{i-1} a_{ij}\Phi_j\ , \quad (6)$$

Where

$$a_{ij}=\langle F_i,\Phi_j\rangle/\langle\Phi_j\Phi_j\rangle,$$

The functions Φ_i can be normalized by using

$$\hat{\Phi}_i=\Phi_i/||\Phi_i|| \quad (7)$$

In evaluating the inner products, we have found the following result given by Singh and Tyagi [6] extremely useful

$$\int_R\int x^p y^q u^r dxdy=a^{p+1}b^{q+1}\frac{\Gamma\frac{p+1}{2}\Gamma\frac{q+1}{2}\Gamma r+1}{\Gamma\frac{p+q}{2}++2+r} \quad (8)$$

Where p and q are non-negative even integers r+1 > 0. If p and q are odd, the integral vanishes. It helps us to express the a_{ij} in closed form.

2.2 Rayleigh Ritz Method

We assume the deflection W(x, y) of the form by eliminating the time variable,

$$W(X,Y)=\sum_{j=1}^{N} c_j\,\Phi_j(x,y). \quad (9)$$

We substitute this in the Rayleigh quotient

$$\omega^2=\frac{D}{h\rho}\frac{\int_R\int[(\nabla^2W)^2+2(1-\upsilon)\{W_{xy}^2-W_{xx}W_{yy}\}]\,dx\,dy}{\int_R\int W^2\,dx\,dy} \quad (10)$$

where

$$D=Eh^3/(12(1-v^2))=\text{flexural rigidity} \quad (11)$$

E = Young's modulus, h = thickness of the plate

v = Poisson's ratio , p = mass density

ω = frequency, **∇** = Laplace operator.

After minimizing ω^2 as a function of the coefficients c_j and simplifying, we get the following standard problem

$$\sum_{j-1}^{n}(a_{ij}-\lambda^2 b_{ij})c_j\ ;\ i=1(l)n; \quad (12)$$

Where

$$\lambda^2=\frac{a^4\omega^2\ \rho h}{D}, \quad (13)$$

$$a_{ij}=\int_R\int\left[\Phi_i^{xx}\Phi_j^{xx}+\Phi_i^{yy}\Phi_j^{yy}+\upsilon(\Phi_i^{xx}\Phi_j^{yy}+\Phi_i^{yy}\Phi_j^{xx})+2(1-\upsilon)\Phi_i^{xy}\Phi_j^{xy}\right]dx\,dy \quad (14)$$

and

$$b_{ij}=\int_R\int\Phi_i\,\Phi_j\,dxdy \quad (15)$$

3 Computational Analysis

For computational analysis we have followed the approach used by T V Lakshmi Reddy [7]. Modeling of Ellipse is carried

out using Solid works with the same dimensions of that in analytical method. Model is imported to Hypermesh. Tetrahedral meshing is used and applied boundary conditions for clamping condition as shown in figure 1. Finally modal analysis is carried out using Ansys. The first three values of natural frequencies and mode shapes are obtained from Computational analysis. Mode shape for third mode for steel elliptical plate is shown in figure 2.

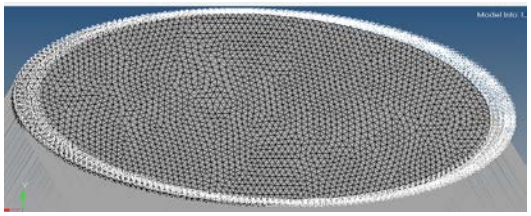


Fig 1 Meshing and BC's applied for Elliptical plate

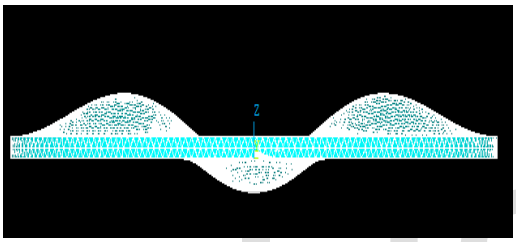


Fig 2 Mode Shape for third mode for Steel

4 Results and Discussions

Analytical analysis results: First three natural frequencies of elliptical plates for both Steel and Aluminium are calculated. The aspect ratio is taken as m $=\frac{b}{a}=0.5$, where b = length of minor axis, a = length of major axis. Results comparison for both steel and aluminum is shown in Table 1

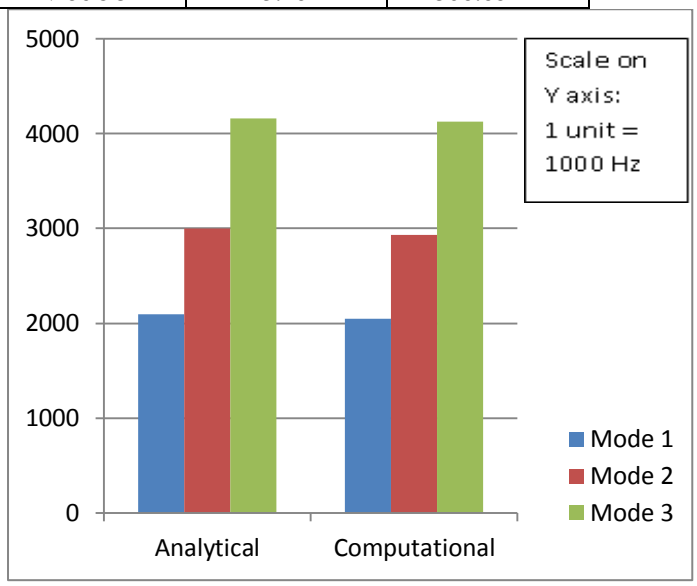
Table 1 Analytical Results

	Materials	
	Steel	Aluminium
Mode 1	2093.13 Hz	2148.05 Hz
Mode 2	2995.73 Hz	3078.36 Hz
Mode 3	4157.61 Hz	4343.72 Hz

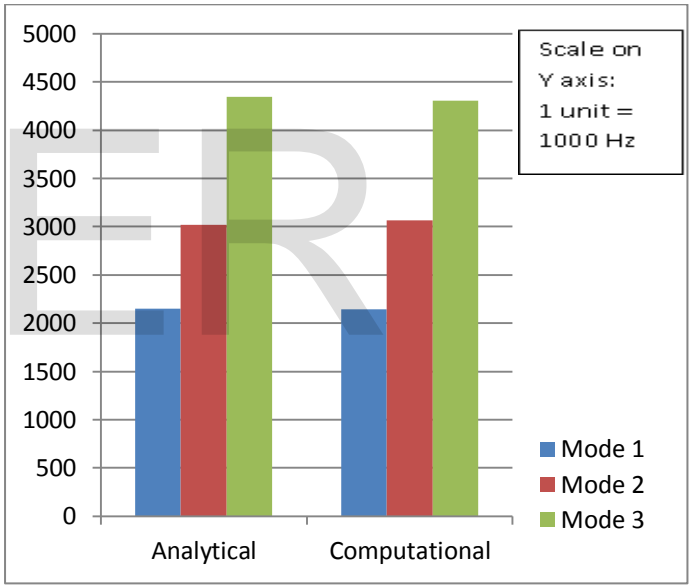
Computational analysis results: Natural Frequency values from computational analysis for both Steel and Aluminium are listed in Table 2.

Table 2 Computational Results

	Materials	
	Steel	Aluminium
Mode 1	2050.90 Hz	2145.91 Hz
Mode 2	2932.71 Hz	3064.11 Hz
Mode 3	4125.16 Hz	4306.68 Hz



Graph 1 Comparison of analytical and computational results for Steel



Graph 2 Comparison of analytical and computational results for Aluminium

We can infer from Graph 1 and Graph 2 that Analytical results are in close agreement with the computational results for different material properties. The percentage of error for computational analysis in comparison with analytical analysis is around 3% to 8%.

From Table 1 and Table 2, it can be observed that the Natural frequency values for Aluminium plate are slightly greater than those of steel plate and they differ by 2% to 4%.

5 Conclusion

Vibration analysis on elliptical plates for different materials is carried out using Analytical and Computational Methods. Fully clamped boundary condition is selected for vibration analysis. The first three modes of frequencies are calculated and compared between analytical and computational methods. The results from computational analysis are in close comparison with the analytical results. The Natural frequency values for Aluminium plate are higher compared to Steel, even though the flexural rigidity value of Aluminium is less compared to Steel. For lightweight applications Aluminium elliptical plate is a better selection over Steel, barring the strength.

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